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Abstract

This paper examines the impact on unemployment, unemployment distribution, wages and welfare of Youth Unemployment Programmes (YUPs). The aim of YUP is to increase the number of young people acquiring skills. We assume that the YUPs are a complete success and consequently analyse what happens when the number of skilled workers increases relatively to the number of unskilled workers. The results depend on the productivity of the skilled workers when employed in the 'unskilled sector' relatively to the productivity of the unskilled worker.

Keywords: Skill, unemployment, search
JEL classifications: J18, J38, J68

1. Introduction

Reduction of high European unemployment rates has been of great concern to the politicians in the European countries throughout the nineties. In many countries, this concern has resulted in policy attempts in order to fight the problem. Different policies have been used and a shift from passive unemployment policy, generally, simply in the form of unemployment insurance paid to unemployed workers towards active labour market policy has been initiated in many countries. For example, Nickell and Van Ours (2000) analyses the Dutch and British cases. Both countries have experienced a large drop in unemployment during the nineties. Denmark has experienced an even larger fall in unemployment during the same period. We therefore give a short description of the policy implemented

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in Denmark in the nineties, even though some of the same elements may be found in the policy conducted in other countries.

In Denmark in 1996 a reform directed towards the unemployed, low-educated youth was implemented, the Youth Unemployment Programme (YUP). The aim of this reform was to improve the employment possibilities for unemployed, low-educated youth by motivating them to undertake an education.[1] The motivation was the fact that the unemployment rate of skilled workers was (and still is) less than the unemployment rate of unskilled workers in Denmark, and this ordering has been present in more than twenty years. Groes and Holm (1999) analyse the unemployment rates in Denmark dependent upon educational level for the period 1980-1998. The unemployment rate of skilled workers is below the average. This ordering has been the same throughout the period, and there is evidence of further polarization: the unemployment rate of non-skilled workers keeps increasing, whereas they expect bottleneck problems for higher educated workers.

In this paper we analyse the long-term effects on unemployment, wages and welfare of the YUP. Furthermore, we consider distribution effects of the YUP. We set up a search model distinguishing between skilled and unskilled workers. We do not model the decision to acquire skills, rather we assume that an exogenous fraction of the labour force has already acquired skills.[2] Consequently, workers are either skilled or unskilled. There are two sectors, a skilled sector where there is a minimum skill requirement and an unskilled sector without any skill requirements. Unskilled workers cannot be hired in the skilled sector whereas skilled workers can be hired in both the unskilled and the skilled sector. This definition of skills enables us to distinguish between skilled jobs and unskilled jobs and skilled workers and unskilled workers. We assume that skilled workers search for jobs in both sectors as the value of employment in the unskilled sector is higher than the value of being unemployed. The assumption that workers search for jobs in both sectors is also found in Gautier (2001) and Albrecht and Vroman (2000). Furthermore it is in accordance with real life behaviour of skilled workers in Denmark. In 1998, 30 percent of the workers who are employed in the unskilled sector are skilled workers according to the Danish Ministry of Labour (2000). We assume that workers can direct their search, i.e., they know whether they apply for a skilled job or an unskilled job. Firms, however, do not know ex ante which type of workers applies for the vacancy. The motivation behind these assumptions is that it is not difficult for a firm to inform about the skill requirements for a job, but they cannot ex ante exclude some workers from applying for the job.

[1] Young *insured* persons, less than 25 years, without any formal education beyond secondary school, and who have been unemployed for 6 months during the last 9 months, are given an offer of 18 months specially designed vocational education. This offer contains an incentive to undertake ordinary education or to find a job since unemployment benefits are cut by 50 percent while in the special education programme. Refusal to participate in the special education programmes is followed by a sanction, it will result in a total loss of unemployment benefits.

[2] See Filges and Larsen (2001) for an analysis of the decision to acquire skills.

Consequently, skilled workers search for jobs in both sectors whereas unskilled workers, knowing they are unemployable in the skilled sector, search for jobs in the unskilled sector only. The wage paid in the skilled sector is higher than the wage offered in the unskilled sector. This implies that skilled workers prefer to work in the skilled sector and consequently, they search for jobs in the skilled sector while employed in the unskilled sector.[3]

We assume that the YUP is a complete success in the sense that it actually increases the relative number of workers acquiring skills. Consequently, we analyse the long-term effect of the YUP by considering an *exogenous* increase in the number of skilled workers relative to the number of unskilled workers. Usually, in standard search models, the unemployment *rate* of a specific worker group is not affected by the number of workers in the group. In the present model, however, a larger fraction of skilled workers implies that the unemployment rate of skilled workers as well as the unemployment rate of unskilled workers are affected. The results depend on the marginal productivity of the skilled worker while employed in the unskilled sector relatively to the productivity of the unskilled worker.

If the marginal productivity of a skilled worker employed in the unskilled sector relative to the marginal productivity of an unskilled worker is lower than a threshold, β^*, where in general β^* is greater than 1, then the effects are as follows. A larger fraction of skilled workers implies that the unemployment rate of skilled workers as well as the unemployment rate of unskilled workers increases. The positive effect on the unemployment rates is due to the assumption that unemployed skilled workers search for jobs in both the skilled and the unskilled sector. The YUP also induces a negative effect on the total unemployment rate. This results from the movement of workers from the high unemployment group, the unskilled workers, to the low unemployment group, the skilled workers. Hence, the net effect of the YUP on the total unemployment rate is ambiguous. Similarly, there is both a negative and a positive welfare effect. Increasing the relative number of skilled workers implies moving workers from the group giving rise to the lowest welfare to the group of workers giving rise to the highest welfare, thereby inducing an increase in total welfare. However, the YUP has a negative impact on the total welfare *rate* in the economy. Consequently, the total welfare effect of the YUP is ambiguous. Considering wage effects we show that wages in the unskilled sector decreases due to the YUP. Concerning distribution effects, we show that the YUP induces a more unequal wage and unemployment distribution. Initially, the wage rate in the skilled sector is higher than wages in the unskilled sector. The YUP worsens this unequal wage distribution. Likewise, unemployment of the unskilled workers is initially higher than unemployment of the skilled workers. The YUP worsens the unequal unemployment distribution too.

When the marginal productivity of the skilled worker employed in the unskilled sector crosses the threshold β^* then these findings are somewhat modified.

[3] This information structure is also found in Gautier (2001).

In this case, we may see a decrease in wage dispersion and the unemployment rates of both skilled and unskilled workers as well as an increase in the welfare rate of the economy.

Our contribution should be compared to two related papers. Gauthier (2001) has been written parallel with our paper and do have some similarities to our paper. However, in his paper wages are only functions of exogenous parameters. Our paper has wages as functions of transition rates as these are important for worker's fall back position. We can therefore study impacts of a change in the fraction of skilled workers on wages and wage distribution. Furthermore, we can consider impacts on welfare. Saint-Paul (1996) perform a similar experiment as we do, but in a more restrictive model, in the sense that there is a fixed number of jobs in the economy and workers employed in the unskilled sector do not search on-the-job.

The paper is organized as follows. The model is presented in Section 2. In Section 3, the equilibrium is described and Section 4 considers the effects of YUP. The last section concludes.

2. The model

In this section we set up the model to analyse the long-term effects on unemployment, wages and welfare of YUP. We do not model the decision to acquire skills, rather we assume that an exogenous fraction of the labour force has already acquired skills. Consequently, the labour force consists of workers who are either skilled or unskilled. In order to focus on the long term impact of the YUP, we analyse the consequences of an exogenous increase in the number of skilled workers.

There are two sectors, sector 1 is the skilled sector and supplies jobs only to skilled workers. Sector 2, is the unskilled sector and supplies jobs to both skilled and unskilled workers. The marginal product in sector 1 is higher than in sector 2. Skilled workers, however, prefer to work in sector 1 as the marginal product is higher than in sector 2 and consequently the wage paid is also higher than the wage offered in sector 2.

2.1. Workers

Let $\Gamma_i^{E_j}$ and Γ_i^U denote the expected present values of lifetime utilities of being employed and unemployed, respectively. Subscript $i = S, N$ denotes skilled respectively unskilled workers and $j = 1, 2$ denotes which sector the worker is employed in. The lifetime utilities of a skilled worker are then:

$$r\Gamma_S^{E_1} = w_s^1 - t + q(\Gamma_S^U - \Gamma_S^{E_1}), \tag{2.1}$$

$$r\Gamma_S^U = b - t + p_1(\Gamma_S^{E_1} - \Gamma_S^U) + p_2\left(\Gamma_S^{E_2} - \Gamma_S^U\right), \tag{2.2}$$

$$r\Gamma_S^{E_2} = w_s^2 - t + p_1\left(\Gamma_S^{E_1} - \Gamma_S^{E_2}\right) - q\left(\Gamma_S^{E_2} - \Gamma_S^{U}\right), \tag{2.3}$$

where b is the unemployment insurance, r is the discount rate and q is an exogenous fraction of currently employed workers leaving their jobs. The wage rate of a skilled worker working in sector 1 is denoted w_s^1, and w_s^2 is the wage rate of a skilled worker employed in sector 2. t is a lump sum tax paid by all workers. p_1 is the transition rate of skilled workers into employment in sector 1 and p_2 is the transition rate of workers into employment in sector 2. Note that unemployed skilled workers search for a job in both sector 1 and sector 2, and that skilled workers employed in sector 2 search for a job in sector 1.

The lifetime utilities of unskilled workers are:

$$r\Gamma_N^{E_2} = w_n^2 - t + q(\Gamma_N^{U} - \Gamma_N^{E_2}), \tag{2.4}$$
$$r\Gamma_N^{U} = b - t + p_2(\Gamma_N^{E_2} - \Gamma_N^{U}), \tag{2.5}$$

where w_n^2 is the wage rate of an unskilled worker employed in sector 2. Note that the transition rates of skilled and unskilled workers into employment in sector 2 are equal. Hence, as argued below, we assume that skilled and unskilled workers compete for jobs in sector 2 on equal terms.

2.2. Firms

Firms supply jobs dependent upon the wage and their hiring costs. Firms supply one job each. Let y be the marginal product of a skilled worker in sector 1, ay, $a < 1$, where $ay > b$, is the marginal product of an unskilled worker in sector 2, finally, $a\beta y$ is the marginal product of a skilled worker working in sector 2, $a\beta < 1$ and $a\beta y > b$. Hence, we have that $b < \min(a\beta y, ay)$. Note, that we assume that the skills of skilled workers do not imply that they necessarily are able to produce more than unskilled workers when working in the unskilled sector. Only when the skilled workers are employed in the skilled sector do they make use of their skills and consequently are able to produce at 'their maximum'. We do not restrict β to be larger or smaller than one, only less than $\frac{1}{a}$. The empirical evidence on this issue is mixed (see Büchel 2000 for a survey) and the intuition is as follows. If $\beta = 1$, that is, a skilled worker employed in sector 2 has the same marginal productivity as an unskilled worker employed in sector 2. This assumption implies that for example, when working as a cleaner, it does not matter for the marginal product whether you are able to control a crane or not. The number of fluff removed in a day is the same. On the other hand, when controlling a crane it surely matters for the marginal product whether you are trained to do that or not. If $\beta > 1$, the skilled worker is more productive than the unskilled workers, while employed in sector 2. This may happen, for example, in the tourist sector, where some knowledge of language may be an advantage even in unskilled kind of jobs, or it may happen if skilled workers have a higher

level of job satisfaction or better health status. The case of $\beta < 1$, corresponds to the situation where skilled workers when performing unskilled kind of jobs are so bored and frustrated that it negatively influences their level of job satisfaction and health status and consequently lowers their productivity.[4]

The expected present values of a filled job in sector j, $j = 1, 2$, Γ_j^J, and of a vacant job, Γ_j^V, are determined by the equations:

$$\begin{aligned}
r\Gamma_1^J &= y - w_s^1 - q(\Gamma_1^J - \Gamma_1^V), && (2.6)\\
r\Gamma_1^V &= \lambda_1 \left(\Gamma_1^J - \Gamma_1^V\right) - k, && (2.7)\\
r\Gamma_2^J \left(w_n^2\right) &= ay - w_n^2 - q(\Gamma_2^J \left(w_n^2\right) - \Gamma_2^V), && (2.8)\\
r\Gamma_2^J \left(w_s^2\right) &= ay\beta - w_s^2 - (p_1 + q) \left(\Gamma_2^J \left(w_s^2\right) - \Gamma_2^V\right), && (2.9)\\
r\Gamma_2^V &= \lambda_2(\lambda_N \Gamma_2^J \left(w_n^2\right) + (1 - \lambda_N) \Gamma_2^J \left(w_s^2\right) - \Gamma_2^V) - k, && (2.10)
\end{aligned}$$

where λ_j, $j = 1, 2$, is the firm's transition rate, and λ_N is the probability that the worker searching for a job in sector 2 is unskilled. The direct cost associated with job supply is given by k. Free entry implies that jobs are supplied as long as it is profitable, i.e. until $\Gamma_i^V = 0$. Using this condition and combining equations (2.6)-(2.7), respectively (2.8)-(2.10) gives two equations to determine the firms' transition rates:

$$\begin{aligned}
-k + \lambda_1 \frac{y - w_s^1}{r + q} &= 0, && (2.11)\\
-k + \lambda_2 \left(\lambda_N \left(\frac{ay - w_n^2}{r + q}\right) + (1 - \lambda_N) \left(\frac{ay\beta - w_s^2}{r + q + p_1}\right)\right) &= 0. && (2.12)
\end{aligned}$$

2.3. Matching and Unemployment

The work force is divided into two groups according to skills. We normalize the total labour force to one. The number of workers who have acquired skills is given by Λ and the number of unskilled workers is $1 - \Lambda$. E_i^j, $i = S, N$, and $j = 1, 2$, denotes the number of workers of type i employed in sector j, and U_i, $i = S, N$, denotes the number of unemployed workers of type i. The flows of workers is illustrated in Figure 1.

[4] A number of US studies show that overqualified workers have a lower level of job satisfaction and more health problems and consequently have higher rates of shirking or absenteeism than correctly allocated workers, see Büchel 2000.

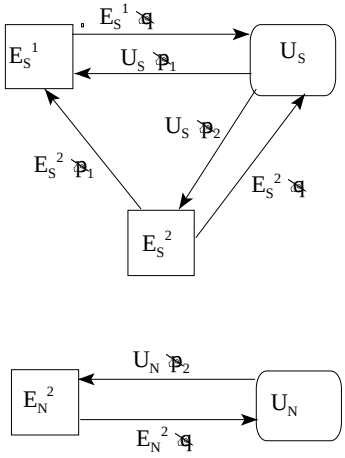

Figure 1: Labour Market Flows.

In steady state, inflows are equal to outflows. The equations determining equilibrium unemployment for unskilled and skilled workers, employment for skilled workers employed in sector 1 and in sector 2 and total numbers are:

$$\begin{aligned} p_2 U_N &= q E_N^2, \\ (p_1 + p_2) U_S &= q \left(E_S^1 + E_S^2 \right), \\ p_1 \left(U_S + E_S^2 \right) &= q E_S^1, \\ p_2 U_S &= (q + p_1) E_S^2, \\ U_N + E_N^2 &= 1 - \Lambda, \\ U_S + E_S^1 + E_s^2 &= \Lambda. \end{aligned}$$

The number of workers in each of the categories is then given by:

$$E_S^1 = \frac{p_1}{q + p_1} \Lambda, \tag{2.13}$$

$$E_S^2 = \frac{p_2}{q + p_1} U_S, \tag{2.14}$$

$$U_s = \Lambda \frac{q}{q + p_1 + p_2}, \tag{2.15}$$

$$E_N^2 = (1 - \Lambda) \frac{p_2}{q + p_2}, \tag{2.16}$$

$$U_N = (1 - \Lambda) \frac{q}{q + p_2}. \tag{2.17}$$

The unemployment rates are both increasing in the separation rate, q, and decreasing in the transition rate p_2. In addition the unemployment rate of skilled workers decreases with p_1. The employment rates all decrease with the separation

rate, q. In addition, the employment rate of skilled workers in sector 1 increases with the transition rate p_1 and the employment rate of unskilled workers in sector 2 increases with the transition rate p_2. The employment rate of skilled workers employed in sector 2 increases with the transition rate p_2 and decreases with the transition rate p_1.

The employment prospects of skilled workers are better than for unskilled workers, see equation (2.15) and (2.17):

$$\frac{U_N}{1-\Lambda} > \frac{U_S}{\Lambda}.$$

Furthermore, the rate of skilled workers employed in skilled jobs is higher than the rate of unskilled workers employed in unskilled jobs if the transition rate into jobs in sector 1 is higher than the transition rate into jobs in sector 2:

$$p_1 > p_2 \Rightarrow \frac{E_S^1}{1-\Lambda} > \frac{E_N^2}{\Lambda}. \tag{2.18}$$

We will only consider parameter values for which the employment rate of skilled workers in skilled jobs is higher than the employment rate of unskilled workers in unskilled jobs. This is definitely the most realistic situation. Groes and Holm (1999) find that the unemployment rate for unskilled workers in Denmark has been above average, whereas the unemployment of skilled workers has been below average, throughout the period 1980-1998. Furthermore, the philosophy behind the YUP is to increase the employment prospects of young people by motivating them to undertake an education. If the inequality in (2.18) is reversed the YUP does not make any sense.

The number of matches formed in sector 1, respectively, sector 2, is given by the matching functions:

$$x_j(V_j, fS_j) = \sqrt{V_j}\sqrt{fS_j}, \;\; j = 1, 2, \tag{2.19}$$

where V_j is the number of vacancies in sector j, and fS_j is the total number of workers searching for a job in sector j measured in efficiency terms. f is a measure of the workers search efficiency. Note that the number of matches has positive first order derivatives in V_j and fS_j, negative second order derivatives, positive cross partial derivatives and is homogenous of degree one in V_j and fS_j. Pissarides 86 and Blanchard and Diamond 89 provide empirical justification for the Cobb-Douglas matching function with equal exponents.

The workers' transition rate, p_j, $j = 1, 2$, is equal to the number of matches divided by the number of searchers. The firms' transition rate is equal to the number of matches divided by the number of vacancies. We measure the transition rates in terms of labour market tightness, θ_j. Labour market tightness is a measure of how tight the market is, and is given by vacancies relative to the number of searchers in efficiency terms. The transition rates become:

$$p_j = f\sqrt{\theta_j},\ \lambda_j = \left(\sqrt{\theta_j}\right)^{-1},\ \theta_j = \frac{V_j}{fS_j},\quad j = 1, 2. \tag{2.20}$$

Having determined the equilibrium unemployment and employment rates and having found expressions for the transition rates, we can rewrite the equilibrium conditions for the transition rates, equation (2.11) and (2.12). By use of equation (2.13)-(2.16) we can express the probability that a worker applying for a job in the unskilled sector is unskilled, λ_N, in terms of $p_j,\ \ j = 1, 2$:

$$\lambda_N = \frac{U_N}{U_N + U_S} = \frac{1}{1 + \frac{\Lambda}{1-\Lambda}\frac{p_2+q}{p_1+p_2+q}} \tag{2.21}$$

By use of (2.20) we get:

$$\lambda_j = \frac{f}{p_j},\ j = 1, 2 \tag{2.22}$$

Hence equation (2.11) and (2.12), giving the equilibrium conditions for the transition rates p_1 and p_2 respectively, can be rewritten to:

$$\Phi = -k + \frac{f}{p_1}\left(\frac{y - w_s^1}{r+q}\right) = 0, \tag{2.23}$$

$$\Psi = -k + \frac{f}{p_2}\left(\frac{\frac{ay-w_n^2}{r+q} - \frac{ay\beta - w_s^2}{r+q+p_1}}{1 + \frac{\Lambda}{1-\Lambda}\frac{p_2+q}{p_1+p_2+q}} + \frac{ay\beta - w_s^2}{r+q+p_1}\right) = 0. \tag{2.24}$$

2.4. Wage determination

We assume that when a firm and a worker meet, they bargain over the wage with an equal bargaining power. Wages then split the match surplus between the firm and the worker equally. The firms have the option of hiring the worker with an expected return of respectively Γ_1^J, $\Gamma_2^J(w_s^2)$ and $\Gamma_2^J(w_n^2)$, or not hiring the worker and pose the vacancy again with expected returns of Γ_1^V and Γ_2^V, respectively. The worker's expected return from acceptance is given by the expected returns from holding a job, $\Gamma_S^{E_1}$, $\Gamma_S^{E_2}$ and $\Gamma_N^{E_2}$. The return from not accepting the job is for unemployed skilled and unskilled workers given by Γ_S^U respectively Γ_N^U. Skilled workers employed in sector 2 will generally have the option of returning to there current jobs, before agreeing to form a new match. However, this option will not be available once the new job is accepted, hence the worker's only alternative after the new match is formed is unemployed search. If we assume that wages can be renegotiated at any time, the expected net surplus from accepting the job is the value of holding the job minus the value of being unemployed: $\Gamma_S^{E_1} - \Gamma_S^U$.[5]

[5] This assumption is made in Pissarides (1994), implying that the wage of a skilled worker does not depend on whether he is unemployed or is employed in sector 2.

Hence, using the equilibrium conditions $\Gamma_1^V = 0$ and $\Gamma_2^V = 0$, the wages, w_s^1, w_s^2, w_n^2, are determined by the equations (see Pissarides (90)):

$$\begin{aligned}
\Gamma_1^J - \left(\Gamma_S^{E_1} - \Gamma_S^U\right) &= 0, \\
\Gamma_2^J(w_s^2) - \left(\Gamma_S^{E_2} - \Gamma_S^U\right) &= 0, \\
\Gamma_2^J(w_n^2) - \left(\Gamma_N^{E_2} - \Gamma_N^U\right) &= 0.
\end{aligned}$$

Using equation (2.1)-(2.5) and equation (2.6)-(2.10) the first order conditions can be rewritten:

$$\frac{y - w_s^1}{r+q} - \frac{w_s^1 - b - p_2 \frac{w_s^2 - b}{r+p_1+p_2+q}}{r+q+p_1} = 0, \tag{2.25}$$

$$\frac{ay\beta - w_s^2}{r+q+p_1} - \frac{w_s^2 - b}{r+p_1+p_2+q} = 0, \tag{2.26}$$

$$\frac{ay - w_n^2}{r+q} - \frac{w_n^2 - b}{r+q+p_2} = 0. \tag{2.27}$$

The wages may be written as:

$$\begin{aligned}
w_s^1 &= x_1 y + (1 - x_1)\, b + x_3 \left(a\beta y - b\right), \\
w_s^2 &= x_0 a\beta y + (1 - x_0)\, b, \\
w_n^2 &= xay + (1 - x)\, b,
\end{aligned}$$

where $x_1 = \frac{r+q+p_1}{p_1+2(r+q)}, x_0 = \frac{r+q+p_1+p_2}{p_2+2(r+q+p_1)}, x = \frac{r+q+p_2}{p_2+2(r+q)}, x_3 = (1 - x_1)\frac{p_2}{p_2+2(r+q+p_1)}$. It is shown in Appendix A that the wage ordering is as follows. In general we have that $w_s^1 > w_s^2$ and $w_s^1 > w_n^2$. Furthermore,

$$w_n^2 \gtreqless w_s^2 \text{ for } \beta \lesseqgtr \beta_1,$$

where β_1 is defined by the equation:

$$\frac{a\beta_1 y - b}{ay - b} = \frac{(p_2 + r + q)\left(p_2 + 2\left(r + q + p_1\right)\right)}{\left(p_2 + 2\left(r + q\right)\right)\left(p_1 + p_2 + r + q\right)}, \tag{2.28}$$

and $\beta_1 > 1$.

The wage rate in sector 1 is higher than wages in sector 2 as the marginal productivity is the highest in sector 1.[6] Concerning the relative wages of the skilled and unskilled workers employed in sector 2 we have two different cases, dependent upon their relative productivities, i.e. depending on β. Consider first

[6] This ordering of wages is in accordance with the Danish skilled versus unskilled wages. Based on numbers from 1998, provided by the Danish Employers Federation, Ministry of Education, Ministry of Labour and Ministry of Social Affairs.

the case where $\beta < \beta_1$. The wage of an unskilled worker is higher than the wage of a skilled worker employed in sector 2 if the productivity of the skilled workers employed in sector 2 is lower than or not too much higher than the productivity of an unskilled worker. The intuition is as follows. A skilled worker's expected return of not accepting a job in sector 2 is higher than the unskilled worker's expected return, $\Gamma_S^U > \Gamma_N^U$, implying a higher wage demand of the skilled workers. On the other hand, as skilled workers search for a job in sector 1 while employed in sector 2, the firm has to be compensated for the higher risk of being separated from the worker. The last effect dominates, implying unskilled workers receive a higher wage than skilled workers when employed in sector 2.

Consider now the case, $\beta > \beta_1$, that is, the productivity of a skilled worker employed in sector 2 is much larger than the productivity of an unskilled worker. This condition is more likely to be satisfied, the higher the level of unemployment insurance. In this case, the relatively higher productivity of the skilled worker more than compensates for the higher risk of separation. Hence, the skilled worker employed in sector 2 receives a higher wage than the unskilled worker.

3. Equilibrium

The equilibrium is found by inserting the equilibrium wages, equation (2.25)-(2.27), into equation (2.23) and (2.24), giving the equilibrium transition rates, p_1 as a function of p_2, $p_1(p_2)$, and p_2 as a function of p_1, $p_2(p_1)$, respectively. Only the equilibrium transition rate, p_2, directly depends upon the number of skilled workers, Λ. How this transition rate is affected depends on the relative productivities of the workers employed in sector 2 relatively to the separation rate concerning a specific sector 2− worker pair. In general, the impact on the transition rate facing workers applying for jobs in sector 2 depends on the difference between the firm-values attached to the two types of workers. The value attached to the skilled worker employed in sector 2 depends positively on the worker's productivity and negatively on the worker's transition rate into employment in sector 1, p_1:

$$\Gamma_2^J\left(w_n^2\right) - \Gamma_2^J\left(w_s^2\right) = \frac{ay - w_n^2}{r+q} - \frac{a\beta y - w_s^2}{r+q+p_1}.$$

The higher this transition rate, p_1, is, the shorter the match, and hence, the lower the expected return for the sector 2 firm when employing a skilled worker.

The equation:

$$\frac{ay - w_n^2}{r+q} - \frac{a\beta y - w_s^2}{r+q+p_1} = \frac{ay-b}{p_2 + 2(r+q)} - \frac{a\beta y - b}{p_2 + 2(r+q+p_1)} = 0,$$

defines β^*, where $\beta^* > 1$(See Appendix C).

If the productivity of the skilled worker while employed in sector 1 is not too much higher than the productivity of the unskilled worker, $\beta < \beta^*$, we have that when the number of skilled workers increases, Λ increases, the transition rate into employment in sector 2 decreases, see Appendix C:

$$\frac{\partial p_2}{\partial \Lambda}|_{\beta<\beta^*} = -\frac{\partial \Psi/\partial \Lambda}{\partial \Psi/\partial p_2} < 0.$$

When the relative number of skilled workers increases, the probability that a worker applying for a job in sector 2 is skilled, increases too. The value of filling a job with a skilled worker is less than the value of filling the job with an unskilled worker in sector 2 implying, that fewer vacancies are supplied in the unskilled sector and consequently the transition rate p_2 decreases. Hence, when the number of skilled workers increases relative to the number of unskilled workers, the chance of getting a job in the unskilled sector decreases.

Consider now the case where $\beta > \beta^*$. We show in Appendix C that the transition rate into employment into sector $2, p_2$ is increasing with Λ if $\beta^* < \beta$:

$$\frac{\partial p_2}{\partial \Lambda}|_{\beta>\beta^*} = -\frac{\partial \Psi/\partial \Lambda}{\partial \Psi/\partial p_2} > 0,$$

Concerning the transition rate p_1, it is shown in Appendix C, that p_1 as a function of p_2 is negatively sloped:

$$\frac{\partial p_1}{\partial p_2} \mid p_1(p_2) = -\frac{\partial \Phi/\partial p_2}{\partial \Phi/\partial p_1} < 0.$$

We can go one step further and show that in general the slope lies between zero and minus one:

$$-1 < \frac{\partial p_1}{\partial p_2} \mid p_1(p_2) < 0 \quad \forall \beta > 0. \tag{3.1}$$

The existence of a stable equilibrium requires that: $\frac{dp_2}{dp_1} \mid p_2(p_1) > -1$. In appendix C we show that:

$$\frac{dp_2}{dp_1} \mid p_2(p_1) > -1 \text{ iff } \beta < \hat{\beta},$$

where $\hat{\beta} > \beta^*$. Existence requires that β can not be too large. Consequently, we restrict β to be less than $\min\left(\frac{1}{a}, \hat{\beta}\right)$.

Above we have shown that when the transition rate into employment in sector 2 decreases (increases), the transition into employment in sector 1 increases (decreases). The transition rate, p_1, is not directly dependent upon p_2. The effect works through the wage of workers employed in the skilled sector, w_s^1, see equation (2.23) and (2.25). If the value of unemployment increases due to a higher

transition rate into employment in the unskilled sector, p_2, this tends to increase the wage of skilled workers employed in the skilled sector (it is shown below that $\frac{\partial w_s^1}{\partial p_2} > 0$). The higher wage implies that the value of having a filled job in the skilled sector decreases, corresponding to that less vacancies are supplied, and consequently the transition rate p_1 decreases.

The equilibrium transition rates, p_1 and p_2, for Λ equal to 0.5 respectively 0.8 and $\beta < \beta^*$ are illustrated in Figure 2.

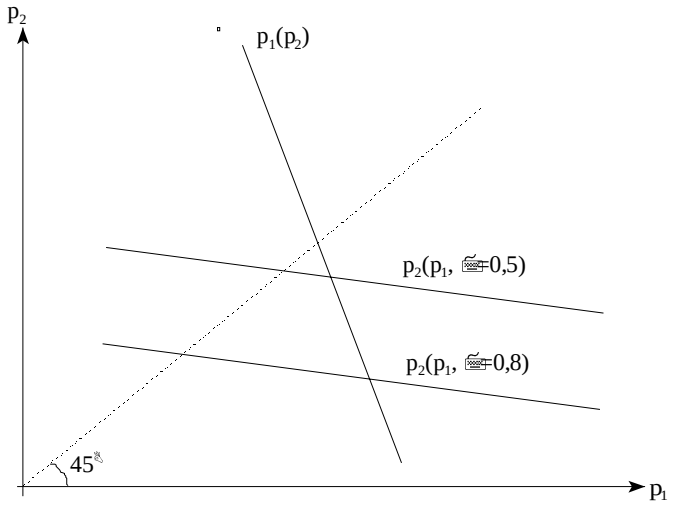

Figure 2: Equilibrium Transition Rates.

The negatively sloped steep line represents p_1 as a function of p_2, given in equation (2.23), and the top line respectively the bottom line, represent p_2 as a function of p_1, given in equation (2.24), when $\Lambda = 0.5$ respectively $\Lambda = 0.8$ and $\beta = 1$.

As shown above, when Λ increases, the curve giving p_2 as a function of p_1 moves downward for $\beta < \beta^*$, implying that p_2 decreases, whereas p_1 increases. For $\beta = \beta^*$ the curve does not move and for $\beta > \beta^*$ the $p_2(p_1)$ curve moves upwards whenever Λ increases. In this case, p_2 increases corresponding to a lower p_1.

4. Effects of YUP

We analyse the impact of YUP by considering an increase in the number of skilled workers, keeping constant the total number of workers, i.e., we consider the effects of increasing the exogenous parameter Λ. In doing so we implicitly assume the best of all worlds, namely that the YUP is a complete success. We examine the impact on wages, wage dispersion, unemployment and welfare.

4.1. Wages

When the number of skilled workers increases, the worker's transition rates are affected, as described above in the equilibrium section. The impact on wages depend on the relative productivities of worker employed in sector 2. In the case where the productivity of a skilled workers employed in sector 2 is not too much higher than the productivity of an unskilled worker, $\beta < \beta^*$, we have the following. The transition rate for workers searching for a job in sector 2, p_2, will decrease and the transition rate for workers searching for a job in sector 1, p_1, increases. As shown in Appendix B the decrease in p_2 corresponds to a decrease in all wages, whereas the increase in p_1 has no impact on wages received by unskilled workers, tend to decrease wages for skilled workers employed in sector 2 and tend to increase wages for skilled workers employed in sector 1. Whenever $\beta > \beta^*$ the results are the opposite. The total impact on wages resulting from an increase in Λ is given by the following proposition:

Proposition 4.1. *When the fraction of skilled workers increases, Λ increases, we have for $\beta < \beta^*$ ($\beta > \beta^*$) that sector 2 wages decrease (increase), $\frac{dw_n^2}{d\Lambda} < 0$ (>0), $\frac{dw_s^2}{d\Lambda} < 0$ (>0) whereas the effect on the sector 1 wage rate is ambiguous, $\frac{dw_s^1}{d\Lambda} \lesseqgtr 0$.*

Proof. Wages change with Λ according too:

$$\frac{dw_n^2}{d\Lambda} = \frac{dx}{dp_2}\frac{\partial p_2}{\partial \Lambda}\left(ay - b\right),$$

$$\frac{dw_s^2}{d\Lambda} = \left(\frac{\partial x_0}{\partial p_2} + \frac{\partial x_0}{\partial p_1}\frac{\partial p_1}{\partial p_2}\right)\frac{\partial p_2}{\partial \Lambda}\left(a\beta y - b\right),$$

$$\frac{dw_s^1}{d\Lambda} = \left(\left(y - b\right)\frac{\partial x_1}{\partial p_1}\frac{\partial p_1}{\partial p_2} + \left(a\beta y - b\right)\left(\frac{\partial x_3}{\partial p_1}\frac{\partial p_1}{\partial p_2} + \frac{\partial x_3}{\partial p_2}\right)\right)\frac{\partial p_2}{\partial \Lambda}.$$

The signs of $\frac{dw_n^2}{d\Lambda}$ and $\frac{dw_s^2}{d\Lambda}$ are equal to the sign of $\frac{\partial p_2}{\partial \Lambda}$, for which we derived conditions in the previous section. As $\frac{dx_3}{dp_2} > 0$, $\frac{dx_1}{dp_1} > 0$ and $\frac{dx_3}{dp_1} < 0$, the sign on the third derivative is in general ambiguous. In Appendix B it is shown that $\frac{\partial x_1}{\partial p_1} + \frac{\partial x_3}{\partial p_1} > 0$. However, the decrease in p_2 has a direct negative impact on w_s^1. As $-1 < \frac{\partial p_1}{\partial p_2} \mid p_1\left(p_2\right) < 0$, we cannot determine which effect will dominate. ■

Consider the case when $\beta < \beta^*$. The shift of workers from the unskilled to the skilled labour force is associated with wage decreases for workers employed in sector 2. This is so as the transition rate for workers into sector 2 decreases, decreasing their bargaining power. The transition rate facing skilled workers concerning their job search into sector 1 increases, tending to increase skilled workers' bargaining power. The impact on sector 1 wages is therefore ambiguous.

Consider next the case where $\beta > \beta^*$. The transition rate into employment in sector 2 increases corresponding to wage decreases for the workers employed in

this sector. Symmetrically we have a decrease in the transition rate into sector 1, again inducing an ambiguous impact on sector 1 wages.

4.2. Wage dispersion

In the previous section we have shown that sector 2 wages decrease (increase) for $\beta < \beta^*$ ($\beta > \beta^*$) and the impact on sector 1 wages is ambiguous, following an increase in the fraction of skilled workers. In this section we evaluate what happens to wage dispersion, both between unskilled and skilled workers and in-between sector 1 and sector 2 workers. Wage dispersion between unskilled and skilled workers employed in sector 2 is defined as

$$WD_2 = w_n^2 - w_s^2 \gtreqless 0 \text{ for } \beta \lesseqgtr \beta_1,$$

where $\beta_1 > 1$ is defined in equation (2.28).

Wage dispersion between skilled workers employed in sector 1 and unskilled workers is positive:

$$WD_{sn} = w_s^1 - w_n^2 > 0.$$

Finally, wage dispersion between skilled workers employed in sector 1 and 2 is positive:

$$WD_{ss} = w_s^1 - w_s^2 > 0.$$

Proposition 4.2. *When the fraction of skilled workers increases, Λ increases, wage dispersion between skilled workers in sector 1 and unskilled workers increases (decreases) for $\beta < \beta^*$ ($\beta^* < \beta < \bar{\beta}$) $\frac{\partial WD_{sn}}{\partial \Lambda} > 0$ $\left(\frac{\partial WD_{sn}}{\partial \Lambda} < 0\right)$, wage dispersion in-between skilled workers increases (decreases) for $\beta < \beta^*$ ($\beta^* < \beta$) $\frac{\partial WD_{ss}}{\partial \Lambda} > 0$ $\left(\frac{\partial WD_{ss}}{\partial \Lambda} < 0\right)$, and the sign of the change in wage dispersion between sector 2 workers is indeterminate.*

Proof. Differentiating wage dispersion with respect to Λ we obtain:

$$\begin{aligned}
\frac{\partial WD_2}{\partial \Lambda} &= \frac{dw_n^2}{d\Lambda} - \frac{dw_s^2}{d\Lambda}, \\
\frac{\partial WD_{sn}}{\partial \Lambda} &= \frac{dw_s^1}{d\Lambda} - \frac{dw_n^2}{d\Lambda}, \\
\frac{\partial WD_{ss}}{\partial \Lambda} &= \frac{dw_s^1}{d\Lambda} - \frac{dw_s^2}{d\Lambda}.
\end{aligned}$$

Substituting for the derivatives gives:

$$\begin{aligned}
\frac{\partial WD_2}{\partial \Lambda} &= \left(\frac{dx}{dp_2}(ay-b) - \frac{dw_s^2}{dp_2} - \frac{dw_s^2}{dp_1}\frac{\partial p_1}{\partial p_2}\right)\frac{\partial p_2}{\partial \Lambda}, \\
\frac{\partial WD_{sn}}{\partial \Lambda} &= \left(\frac{dw_s^1}{dp_1}\frac{\partial p_1}{\partial p_2} + (a\beta y - b)\frac{\partial x_3}{\partial p_2} - \frac{dx}{dp_2}(ay-b)\right)\frac{\partial p_2}{\partial \Lambda}, \\
\frac{\partial WD_{ss}}{\partial \Lambda} &= \left(\frac{dw_s^1}{dp_1}\frac{\partial p_1}{\partial p_2} + (a\beta y - b)\frac{\partial x_3}{\partial p_2} - \frac{dw_s^2}{dp_2} - \frac{dw_s^2}{dp_1}\frac{\partial p_1}{\partial p_2}\right)\frac{\partial p_2}{\partial \Lambda}.
\end{aligned}$$

Using $a\beta < 1, a < 1$, and $\frac{\partial p_1}{\partial p_2} < 0$ it is shown in Appendix D that the sign of the second derivative is positive for $\beta < \beta^*$ and negative for $\beta^* < \beta < \bar{\beta}$, where $\bar{\beta}$ is determined by the equation:

$$\frac{a\beta y - b}{ay - b} = \frac{(p_1 + 2(r+q))(p_2 + 2(r+q+p_1))^2}{2(r+q+p_1)(p_2 + 2(r+q))^2},$$

and the sign of the last derivative is positive for $\beta < \beta^*$ and negative for $\beta > \beta^*$. ■

For $\beta < \beta^*$ we then have the following. Both wages of sector 2 decrease and we cannot determine which of the two wages decreases the most. Wage dispersion in-between skilled workers employed in sector 1 and unskilled workers will increase. Hence, even if the wage rate for skilled sector 1 workers should decrease, it decreases less than the wage rate received by unskilled workers.

Wage dispersion in-between skilled workers employed in the two sectors increases for $\beta < \beta^*$. The shift of workers from the unskilled to the skilled labour force induces a more unequal wage distribution. The intuition is here that both skilled workers negotiating wages in sector 1 and sector 2 are equally affected by the fall in the transition rate into employment in sector 2, whereas the higher transition rate into sector 1 employment tends to increase sector 1 wages. For $\beta > \beta^*$ the transition rate into sector 2 increases with Λ. In this case, wage dispersion in-between skilled workers employed in sector 1 and unskilled workers will decrease if $\beta < \bar{\beta}$ and wage dispersion in-between skilled workers employed in the two sectors decreases.

4.3. Unemployment

Considering unemployment, we first consider the impact on the unemployment rates for skilled and unskilled workers when increasing the number of skilled workers relative to the number of unskilled workers.

Proposition 4.3. *When the fraction of skilled workers increases, Λ increases, unemployment rates for skilled and unskilled workers increase (decrease), $\frac{d(U_S/\Lambda)}{d\Lambda} > 0$ (< 0), $\frac{d(U_N/(1-\Lambda))}{d\Lambda} > 0$ (< 0) for $\beta < \beta^*, (\beta > \beta^*)$.*

Proof. Differentiating the two unemployment rates with respect to Λ we obtain:

$$\frac{d(U_S/\Lambda)}{d\Lambda} = -\frac{U_S\left(\frac{dp_1}{dp_2} \mid p_1(p_2) + 1\right)}{(p_1 + p_2 + q)}\frac{dp_2}{d\Lambda}, \tag{4.1}$$

$$\frac{d(U_N/(1-\Lambda))}{d\Lambda} = -\frac{U_N}{(p_2+q)}\frac{dp_2}{d\Lambda}, \tag{4.2}$$

where we showed above (see equation (3.1)) that:

$$-1 < \frac{\partial p_1}{\partial p_2} \mid p_1(p_2) < 0 \Rightarrow \frac{\partial p_1}{\partial p_2} \mid p_1(p_2) + 1 > 0, \ \forall \beta > 0.$$

■

The net effect on both unemployment rates of increasing the number of skilled workers is positive for $\beta < \beta^*$. The unemployment rate of unskilled workers increases because the value of employing a job with a skilled worker is less than the value of filling the job with an unskilled worker. When the number of skilled workers increases relative to the number of unskilled workers the probability of employing a skilled worker in sector 2 increases. This implies that fewer vacancies are supplied in the unskilled sector and the unemployment rate consequently increases. Symmetrically, when $\beta > \beta^*$ the value of employing a skilled worker is higher in sector 1, implying a higher vacancy supply and thus lower unemployment.

Consider the case where $\beta < \beta^*$. Concerning the unemployment rate of skilled workers, there is a positive as well as a negative effect. The positive effect is directly due to a fall in the number of vacancies supplied in the unskilled sector. The negative effect arises because the transition rate into employment in the skilled sector, p_1, increases. The transition rate, p_1, is not directly dependent upon p_2. The effect works through the wage of skilled workers employed in the skilled sector. As the value of unemployment decreases due to the lower transition rate into employment in the unskilled sector, this tends to decrease the wage of skilled workers employed in the skilled sector (c.i.f. proposition 4.2). The lower wage implies that the value of having a filled job in the skilled sector increases, implying that more vacancies are supplied. The net effect on the unemployment rate is unambiguously positive. When $\beta > \beta^*$, the result is the reverse: higher p_2 and lower p_1 resulting in lower unemployment for skilled workers, U_S.

Consequently, the unemployment rates of both skilled and unskilled workers increase for $\beta < \beta^*$, are unaffected for $\beta = \beta^*$ and decrease for $\beta > \beta^*$ as a result of the larger fraction of skilled workers.

Furthermore, it can be shown that the larger fraction of skilled workers harms the unskilled workers the most when $\beta < \beta^*$, as the increase in the unemployment rate is higher for unskilled workers than for skilled workers. For $\beta > \beta^*$, the decrease in unskilled unemployment is larger than the decrease in skilled unemployment:

$$\begin{aligned} & \frac{d\left(U_S/\Lambda\right)}{d\Lambda} - \frac{d\left(U_N/(1-\Lambda)\right)}{d\Lambda} \\ = \; & q\frac{dp_2}{d\Lambda}\left(\frac{1}{\left(q+p_2\right)^2} - \frac{\frac{dp_1}{dp_2} \mid p_1\left(p_2\right)+1}{\left(q+p_1+p_2\right)^2}\right) \begin{array}{l} <0 \\ =0 \\ >0 \end{array} \; for \left\{ \begin{array}{l} \beta < \beta^* \\ \beta = \beta^* \\ \beta > \beta^* \end{array} \right. \end{aligned}$$

We therefore have the result:

Proposition 4.4. *A larger fraction of skilled workers induces a more unequal unemployment distribution for $\beta < \beta^*$, has no impact for $\beta = \beta^*$ and a more equal unemployment distribution for $\beta > \beta^*$.*

Another interesting thing to consider is the number of skilled workers employed in sector 2. The higher unemployment rate of skilled workers will tend to increase the rate of skilled workers employed in the unskilled sector and the lower vacancy supply in sector 2 will tend to decrease this rate. The net effect can be shown to be negative. We obtain the result.

Proposition 4.5. *The rate of skilled workers employed in sector 2 decreases (unaffected/increases),* $\frac{d\left(E_S^2/\Lambda\right)}{d\Lambda} < 0$ *(*$= 0/ > 0$*) for* $\beta < \beta^*$ *(*$\beta = \beta^*/\beta > \beta^*$*).*

Proof. Differentiating E_S^2/Λ with respect to Λ we have:

$$\frac{d\left(E_S^2/\Lambda\right)}{d\Lambda} = \frac{U_S}{q+p_1}\left(-\frac{p_2\left(\frac{\partial p_1}{\partial p_2} \mid p_1\left(p_2\right)+1\right)\frac{\partial p_2}{d\Lambda}}{q+p_1+p_2} + \frac{\frac{\partial p_2}{d\Lambda}\left(q+p_1\right) - p_2\frac{\partial p_1}{\partial p_2} \mid p_1\left(p_2\right)\frac{\partial p_2}{d\Lambda}}{q+p_1}\right)$$

Rewriting the equation we get:

$$\frac{d\left(E_S^2/\Lambda\right)}{d\Lambda} = \frac{U_S}{\left(q+p_1\right)}\frac{\partial p_2}{d\Lambda}\left(-\frac{\partial p_1}{\partial p_2} \mid p_1\left(p_2\right)\left(\frac{p_2}{q+p_1+p_2}+\frac{p_2}{q+p_1}\right)+\frac{q+p_1}{q+p_1+p_2}\right),$$

showing that the net effect is negative for $\beta < \beta^*$ and positive for $\beta > \beta^*$. ■

The last thing to consider in this section is the impact on the total unemployment rate. The total unemployment rate in the economy is given by:

$$U = \frac{U_S + U_N}{U_S + U_N + E_S^2 + E_S^1 + E_N^2} = U_S + U_N.$$

We can conclude the following:

Proposition 4.6. *The impact on total unemployment following an increase in the number of skilled workers is ambiguous for* $\beta < \beta^*$ *and negative for* $\beta \geq \beta^*$*.*

Proof. The effect of increasing the relative number of skilled workers on total unemployment is:

$$\frac{dU}{d\Lambda} = \left(\frac{U_S}{\Lambda} - \frac{U_N}{1-\Lambda}\right) - \frac{\partial p_2}{d\Lambda}\left(\left(\frac{\partial p_1}{\partial p_2} \mid p_1\left(p_2\right)+1\right)\frac{U_S}{p_1+p_2+q} + \frac{U_N}{p_2+q}\right),$$

where the first parenthesis contain a negative value and the last, squared, parenthesis contain a positive value. As $\frac{\partial p_2}{d\Lambda} < 0$ (≥ 0) for $\beta < \beta^*$ $(\beta \geq \beta^*)$ the net effect is ambiguous for $\beta < \beta^*$ and negative for $\beta \geq \beta^*$. ■

As the unemployment rate of skilled workers is less than the unemployment rate of unskilled workers: $\frac{U_S}{\Lambda} - \frac{U_N}{1-\Lambda} < 0$, the sum of the first two terms is negative. Hence, moving workers from the high unemployment group to the low unemployment group obviously decreases the total unemployment rate. The last

term is the sum of the effects on the skilled and unskilled worker's unemployment rates respectively. As they are both positive for $\beta < \beta^*$, these effects contribute to an increase in the total unemployment rate. In this case, the total effect is thus ambiguous. For $\beta > \beta^*$ both unemployment rate decreases, consequently, unemployment decreases.

To conclude, we have two distinct cases. When $\beta < \beta^*$, what happens when the relative number of skilled workers increases is that, although the chance of getting a job in the skilled sector increases, p_1 increases, it is not enough to outweigh the fall in the transition rate into employment in the unskilled sector, p_2. Hence, the unemployment rates of both skilled and unskilled workers increase. The only positive effect on total employment is the movement of workers from the high unemployment group to the low unemployment group. Furthermore, initially, the employment prospects of an unskilled worker is worse than for a skilled worker. As the unemployment rate of unskilled workers increases more than the unemployment rate of skilled workers, the relative position of an unskilled worker in terms of employment prospects worsen. The larger fraction of skilled workers induces a more unequal unemployment distribution. When $\beta > \beta^*$, the transition rate into sector 2 increases, thereby both unemployment rates and total unemployment decreases. Furthermore, the decrease in unemployment for unskilled workers is larger than the decrease in unemployment for skilled workers, inducing a more equal unemployment distribution. However, in this case there will be a larger fraction of skilled workers employed in sector 1.

4.4. Welfare

In this section we examine the impact on welfare from an increase in the fraction of skilled workers. Given all workers pay a lump sum tax the government budget constraint is given as:

$$b\left(U_s + U_N\right) = t\left(E_S^1 + E_S^2 + E_N + U_S + U_N\right) = t.$$

The welfare function is derived to be:

$$W = W_S^1 + W_S^2 + W_N^1,$$

where

$$\begin{aligned} W_S^1 &= E_S^1 y - V_1 k = \frac{p_1}{q+p_1}\Lambda\left(y - kq\frac{p_1}{f}\right), \\ W_S^2 + W_N^2 &= E_N^2 ay + E_S^2 a\beta y - V_2 k \\ &= \Lambda\frac{qp_2}{\left(q+p_1+p_2\right)\left(q+p_1\right)}\left(a\beta y - k\frac{p_2}{f}\left(q+p_1\right)\right) \\ &\quad + \left(1-\Lambda\right)\frac{p_2}{q+p_2}\left(ay - k\frac{qp_2}{f}\right), \end{aligned}$$

using the government budget constraint to eliminate taxes and unemployment insurance. W_S^1 is welfare associated with sector 1 and $W_S^2 + W_N^2$ is welfare associated with sector 2. Welfare is increasing in employment and productivity and decreasing in vacancy costs. It can be shown that:

$$\frac{W_S^1}{\Lambda} > \frac{W_S^2}{\Lambda}, \frac{W_S^1}{\Lambda} > \frac{W_N^2}{1-\Lambda} \text{ if } p_1 > p_2 \tag{4.3}$$

$$\frac{W_N^2}{1-\Lambda} > \frac{W_S^2}{\Lambda} \text{ if } p_1 > p_2 \text{ and } \beta < \tilde{\beta} \tag{4.4}$$

where $\tilde{\beta} > \beta^*$ is defined by the equation:

$$\begin{aligned} \frac{a\beta y - b}{ay - b} &= \frac{(p_2 + 2(q + p_1))}{(p_2 + 2q)} \frac{p_1 + q}{q} \left(1 + \frac{b}{ay - b}\Omega\right) = 0, \\ \Omega &= \frac{p_1 (p_1 + p_2 + 2q)(p_2 + 2q)}{q (p_2 + q)(p_1 + p_2 + q)(p_1 + q)}. \end{aligned}$$

The "welfare rate" concerning skilled workers employed in the skilled sector, sector 1, is the highest. In general the "welfare rate" concerning unskilled workers is higher than the "welfare rate" concerning skilled workers employed in sector 2. However, when skilled workers employed in sector 2 have a much higher productivity than an unskilled worker, this higher productivity may compensate for the higher risk of separation connected to this worker and consequently the "welfare rate" of unskilled workers is less than the "welfare rate" of skilled workers employed in sector 2.

Considering an increase in the relative number of skilled workers, we can show the following:

Proposition 4.7. *The total welfare effect of increasing the number of skilled workers is ambiguous for $\beta < \beta^*$ and positive for $\beta \geq \beta^*$.*

Proof. Differentiating the welfare equation with respect to the fraction of skilled workers, Λ, we obtain:

$$\frac{W_S^1}{\Lambda} + \frac{W_S^2}{\Lambda} - \frac{W_N^2}{1-\Lambda} + \Lambda \frac{\partial (W_S^1/\Lambda)}{\partial \Lambda} + (1-\Lambda) \frac{\partial (W_N^2/(1-\Lambda))}{\partial \Lambda} + \Lambda \frac{\partial (W_S^2/\Lambda)}{\partial \Lambda}, \tag{4.5}$$

where:

$$\frac{\partial (W_S^1/\Lambda)}{\partial \Lambda} = \frac{dp_1}{dp_2}\frac{dp_2}{d\Lambda}\left(y - \frac{kqp_1}{f} - (q + p_1)\frac{p_1 k}{f}\right)\frac{q}{(q + p_1)^2}, \tag{4.6}$$

$$\frac{\partial (W_N^2/(1-\Lambda))}{\partial \Lambda} = \frac{q}{(q + p_2)^2}\left(ay - \frac{kp_2 q}{f} - (q + p_2)\frac{kp_2}{f}\right)\frac{dp_2}{d\Lambda}, \tag{4.7}$$

$$\frac{\partial (W_S^2/\Lambda)}{\partial \Lambda} = q \frac{\frac{q + p_1 - p_2 \frac{dp_1}{dp_2}}{q + p_1 + p_2}\left(\frac{a\beta y}{q + p_1} - \frac{kp_2}{f}\right) - p_2\left(\frac{a\beta y}{(q + p_1)^2}\frac{dp_1}{dp_2} + \frac{k}{f}\right)}{q + p_1 + p_2}\frac{dp_2}{d\Lambda}. \tag{4.8}$$

Using the equilibrium condition equations (2.23) and (2.24) and the equations for wages in equilibrium (see Appendix A), we can show that:

$$\frac{\partial\left(W_S^1/\Lambda\right)}{\partial\Lambda} = \frac{dp_1}{dp_2}\frac{dp_2}{d\Lambda}\frac{1}{\left(q+p_1\right)^2}\left(bq + \frac{\left(a\beta y - b\right)qp_2}{p_2 + 2\left(q+p_1\right)}\right) \gtreqless 0 \text{ for } \beta \lesseqgtr \beta^*,$$

$$\frac{\partial\left(W_N^2/(1-\Lambda)\right)}{\partial\Lambda} = \frac{bq}{\left(q+p_2\right)^2}\frac{dp_2}{d\Lambda} \lesseqgtr 0 \text{ for } \beta \lesseqgtr \beta^*,$$

$$\frac{\partial\left(W_S^2/\Lambda\right)}{\partial\Lambda} = \frac{q\left(b - p_2\frac{b(q+p_1)^2+\left(3(q+p_1)^2+p_2(p_2+4(q+p_1))\right)a\beta y}{(q+p_1)^2(p_2+2(q+p_1))}\frac{dp_1}{dp_2}\right)}{\left(q+p_1+p_2\right)^2}\frac{dp_2}{d\Lambda} \lesseqgtr 0 \text{ for } \beta \lesseqgtr \beta^*.$$

Using $-1 < \frac{dp_1}{dp_2} < 0$ and $p_1 > p_2$ we can establish that:

$$\Lambda\frac{\partial\left(W_S^1/\Lambda\right)}{\partial\Lambda} + \left(1-\Lambda\right)\frac{\partial\left(W_N^2/(1-\Lambda)\right)}{\partial\Lambda} + \Lambda\frac{\partial\left(W_S^2/\Lambda\right)}{\partial\Lambda} \lesseqgtr 0 \text{ for } \beta \lesseqgtr \beta^*$$

Hence, the total welfare rate effect is negative for $\beta < \beta^*$ and positive for $\beta > \beta^*$.

Furthermore the sum of the first three terms in (4.5) is positive if $p_1 > p_2$:

$$\frac{W_S^1}{\Lambda} + \frac{W_S^2}{\Lambda} - \frac{W_N^2}{1-\Lambda} > 0.$$

■

Welfare associated with sector 1 increases (decreases), whereas welfare associated with sector 2 decreases (increases) for $\beta < \beta^*$ ($\beta > \beta^*$). The total effect of a larger fraction of skilled workers on the welfare rate, the sum of the welfare rates, is negative (positive) for $\beta < \beta^*$ ($\beta > \beta^*$). As welfare associated with sector 1 is higher than welfare associated with sector 2, the movement of people from the group of unskilled workers to the group of skilled workers induces an increase in total welfare. Consequently the effect of YUP on welfare is ambiguous (positive) for $\beta < \beta^*$ ($\beta \geq \beta^*$) in a way similar to the impact of YUP on unemployment.

5. Evaluation

In this papers set up, analytic results concerning the effects on total unemployment and welfare from increasing the relative number of skilled workers can not be derived. Consequently, in this section, simulations are carried out in order to determine the total unemployment and welfare effects.

In the baseline case, the exogenous variables of the model are set at the following values:

$a = 0.75$	$b = 0.45$	$y = 1$	$r = 0.06$	$q = 0.08$	$k = 0.20$	$f = 0.40$	$\Lambda = 0.60$

The marginal productivity of an unskilled worker is set to 75 percent of the marginal productivity of a skilled worker occupying a skilled job. This value is

chosen in order to achieve an unemployment rate dispersion between unskilled and skilled workers in the same order of magnitude as in Denmark in the late nineties. The benefit level is set at $0.6ay = 0.45$, which is in accordance with conditions in Denmark. The relative number of skilled workers in the baseline projection is set at 0.60, which is equal to the level in Denmark in the late nineties. The value of f and the flow cost of keeping a vacancy open are chosen in order to achieve a value of the total unemployment rate in the same order of magnitude as in Denmark in the late nineties. The baseline projection is carried out, assuming that skilled workers are respectively equally, more and less productive when performing unskilled jobs than unskilled workers.

The results of the simulations are shown in Table 1 below.

	$\beta = 1$		$\beta = 1.25$		$\beta = 0.75$		
	$\Lambda = 0.6$	$\Lambda = 0.7$	$\Lambda = 0.6$	$\Lambda = 0.7$	$\Lambda = 0.6$	$\Lambda = 0.7$	
p_1	0.8558	0.8590	0.8041	0.8080	0.8969	0.8984	$\uparrow^*$
p_2	0.5403	0.5077	0.5773	0.5557	0.5092	0.4676	$\downarrow^*$
U	0.0841	0.0798	0.0815	0.0765	0.0866	0.0826	$\downarrow$
$\frac{U_s}{\Lambda}$	0.0542	0.0553	0.0547	0.0554	0.0538	0.0553	$\uparrow^*$
$\frac{U_n}{1-\Lambda}$	0.1290	0.1361	0.1217	0.1259	0.1358	0.1461	$\uparrow^*$
$\frac{U_n}{\Lambda} - \frac{U_s}{1-\Lambda}$	0.0748	0.0808	0.0670	0.0704	0.0819	0.0908	$\uparrow^*$
$\frac{E_s^2}{\Lambda}$	0.0313	0.0299	0.0357	0.0347	0.0281	0.0264	$\downarrow^*$
w_s^1	0.9401	0.9399	0.9437	0.9434	0.9372	0.9371	$\downarrow$
w_s^2	0.6320	0.6304	0.7508	0.7491	0.5173	0.5166	$\downarrow^*$
w_n^2	0.6988	0.6967	0.7010	0.6997	0.6968	0.6938	$\downarrow^*$
$w_s^1 - w_n^2$	0.2413	0.2432	0.2427	0.2437	0.2404	0.2433	$\uparrow^*$
$\frac{W_s^1}{\Lambda}$	0.8832	0.8834	0.8803	0.8805	0.8852	0.8852	$\uparrow^*$
$\frac{W_s^2}{\Lambda}$	0.0156	0.0153	0.0244	0.0240	0.0880	0.0088	$\downarrow^*$
$\frac{W_n^2}{1-\Lambda}$	0.6345	0.6304	0.6384	0.6362	0.6306	0.6245	$\downarrow^*$
W	0.7930	0.8182	0.7982	0.8240	0.7886	0.8132	$\uparrow$

Table 1: Simulation results for β=1, β=0.75, β=1.25.

In the baseline case the value of β^* is equal to 1.602. β^* is the critical value of the relative productivity of skilled workers performing unskilled jobs. Hence, only if skilled workers are 60 percent more productive in unskilled jobs compared to unskilled workers, the results in Table 1 will be reversed (except the effects on total unemployment, U, and total welfare, W).

In the case where $\beta = 1$, that is, all workers are equally productive when performing unskilled jobs, the total unemployment rate is 8.41%, the unskilled unemployment rate is 12.9% and the skilled unemployment rate is 5.42%. The effects of increasing the relative number of skilled workers from 60 percent to 70 percent is the same in all three cases and is shown in the last column. The effect on the endogenous variables where analytic results were obtained in the previous sections is marked with a '*'.

In all three cases the unemployment rates of both skilled and unskilled workers increase as a result of more workers acquiring skills. Unemployment of the unskilled workers is initially higher than unemployment of the skilled workers and the unequal unemployment distribution worsens.

The increased unemployment rates of both worker groups tend to increase the total unemployment rate. However, the negative effect on the total unemployment rate due to the movement of workers from the high unemployment group, the unskilled workers, to the low unemployment group, the skilled workers outweighs the positive effect. Hence, the net effect on total unemployment of YUP is negative.

Similar to the unemployment effect, there is a negative and a positive welfare effect. The YUP has a negative impact on the total welfare rate in the economy. However, welfare associated with the skilled sector is higher than the welfare rate associated with the unskilled sector. Hence, moving workers from the group giving rise to the lowest welfare to the group of workers giving rise to the highest welfare, induces an increase in total welfare.

Wages of the unskilled as well as the skilled sector decrease. Initially, wages in the skilled sector is higher than wages in the unskilled sector. As the skilled sector wage reduction is very small the unequal wage distribution worsens.

As mentioned above the threshold value of β is 1.602 in the baseline case. Hence, if skilled workers are 60 percent more productive in unskilled jobs compared to unskilled workers, the analytic results, marked in Table 1 with a '*', will be reversed. However, β will never reach this level as the upper limit to β is given by $\frac{1}{a} = 1.333\,3$. In order to evaluate how likely it is that β reaches the threshold value β^*, we have changed the parameter values in the baseline projection one by one, until β^* is less than the upper limit $\frac{1}{a}$. The results are shown in Table 2 below:

	β^*	$a\beta^*$	U	$\frac{U_s}{\Lambda}$	$\frac{U_n}{1-\Lambda}$	$\frac{b}{ay}$
baseline	1.602	1.201	0.0841	0.0542	0.1290	0.60
$f = 0.1$	1.002	0.752	0.1750	0.1172	0.2620	0.60
$b = 0.65$	1.281	0.961	0.1363	0.0765	0.2259	0.87
$a = 0.60$	1.588	0.953	0.1093	0.0600	0.1832	0.75
$k = 0.75$	1.485	1.114	−	−	−	0.60
$y = 0.70$	1.273	0.955	0.1597	0.0917	0.2616	0.86
$q = 0.50$	1.296	0.972	0.4524	0.3510	0.6044	0.60
$r = 0.45$	1.312	0.984	0.1206	0.0769	0.1861	0.60

Table 2: Impact on results changing the parameter values one by one.

The parameter change as compared with the baseline case is given in the first column. Note that the upper limit of β^* is given by the inequality $a\beta^* < 1$. In the baseline projection the restriction is binding. Likewise, the restriction is binding when increasing the flow cost of keeping a vacancy to its maximum

$(k < \min(a\beta y, ay))$. The values of unemployment is therefore not shown in this case.

In general, inspection of Table 2 leads to the conclusion that in order to have a non-binding restriction on β^* the equilibrium values of the unemployment rates have to be very high, especially the unskilled unemployment rate has to be unrealistic high. Decreasing the relative productivity of unskilled workers, a, produces the lowest unemployment rates. However, in this case only if skilled workers are more than 58 percent *more* productive performing unskilled jobs compared to unskilled workers, the analytic results, marked in Table 1 with a '*', will be reversed. The realism of this scenario is doubtful.

6. Conclusion

We have analyzed the wage, welfare and distribution effects of YUP by considering an increase in the number of skilled workers, keeping constant the total number of workers.

In our set up, where skilled workers search for jobs in both the skilled and the unskilled sector (sector 1 and sector 2), we have shown that the YUP *do* benefit the workers who participated in the programme. However, in terms of unemployment, wages, wage dispersion and welfare, the results depend on the productivity of a skilled worker while employed in the unskilled sector relatively to the separation rate of this worker. This is the case as the skilled worker while performing an unskilled job, search on the job for a skilled job giving him or her a higher wage. Thereby the skilled worker separates more frequently from an unskilled job than the unskilled worker.

We derive a condition under which both skilled and unskilled workers are better off or worse off in the sense that their probability of obtaining a job in the unskilled sector increases or decreases, respectively. This separates the results into two cases.

In case one, the relative productivity of the skilled worker while performing unskilled kind of work is lower than the threshold β^*, where $\beta^* > 1$. In this case the unemployment rates of both skilled and unskilled workers increase as a result of more workers acquiring skills. Due to their higher expected separation, skilled workers are less attractive for the firm to employ than unskilled workers, even if they have a higher marginal productivity than unskilled workers. Consequently, fewer vacancies are supplied when the relative number of skilled workers increase, thereby decreasing the transition rate for all workers applying for jobs in sector 2. As the unemployment rate of skilled workers is lower than the unemployment rate of unskilled workers, there is a positive as well as a negative effect on total unemployment of the YUP. The increase in the unemployment rates of both worker groups tends to increase the total unemployment rate. The negative effect on the total unemployment rate is due to the movement of workers from the high

unemployment group, the unskilled workers, to the low unemployment group, the skilled workers. Hence, the net effect on total unemployment of YUP is ambiguous, however simulations show that the negative effect due to the movement of workers, dominate.

Similar to the unemployment effect, there is a negative and a positive welfare effect. The YUP has a negative impact on the total welfare rate in the economy. However, welfare associated with the skilled sector is higher than the welfare rate associated with the unskilled sector. Hence, moving workers from the group giving rise to the lowest welfare to the group of workers giving rise to the highest welfare, induces an increase in total welfare. Consequently, the total welfare effect of the YUP is ambiguous. Simulations show that the movement of workers effect dominate in the case of welfare too. Wages of the unskilled sector decrease due to the YUP. Concerning distribution effects, we have shown that the YUP induces a more unequal wage and unemployment distribution. Initially, wages in the skilled sector is higher than wages in the unskilled sector. The YUP worsens this unequal wage distribution. Furthermore, unemployment of the unskilled workers is initially higher than unemployment of the skilled workers. The YUPs worsen the unequal unemployment distribution too.

The other case to consider is when the productivity of skilled workers while performing unskilled work is relatively high. In this case, skilled workers are attractive to the firm and a higher fraction of them implies a higher vacancy supply. The transition rate into sector 2 increases, decreasing both unemployment rates and thereby total unemployment and increasing welfare. Unskilled sector wages increase whereas the impact on the skilled sector wage is still ambiguous. Wage dispersion in-between skilled workers decrease and under a certain range wage dispersion decreases between sector 1 and unskilled workers. The sign of the change of wage dispersion between sector 2 workers is ambiguous.

Which of the two cases is the most appropriate to consider depends on the relevant parameter values of the economy. As skilled workers compared to unskilled workers are more likely to separate from jobs in the unskilled sector, the relative productivity of skilled workers when performing unskilled jobs has to be very high for the skilled workers to be more attractive for the firms than unskilled workers. Simulations show that it is difficult to obtain a reasonable scenario where the threshold value of the relative productivity is below the upper limit ($\beta^* < \frac{1}{a}$). Consequently, in the present set up, it is most likely that we end up in case one, where the number of vacancies supplied in the unskilled sector decreases due to the YUP.

A. Appendix A

By use of the equations (2.25)-(2.27) we obtain:

$$0 = y\left(r+q+p_1\right) - w_s^1\left(p_1+2\left(r+q\right)\right) + b\left(r+q\right) + \frac{p_2\left(w_s^2-b\right)\left(r+q\right)}{r+q+p_1+p_2}, \quad \text{(A.1)}$$

$$0 = ay\beta\left(r+q+p_1+p_2\right) - w_s^2\left(p_2+2\left(r+q+p_1\right)\right) + b\left(r+q+p_1\right), \quad \text{(A.2)}$$

$$0 = ay\left(r+q+p_2\right) - w_n^2\left(p_2+2\left(r+q\right)\right) + b\left(r+q\right). \quad \text{(A.3)}$$

We want to show that the relation between w_s^2 and w_n^2. Rewrite equation (A.2) and (A.3):

$$\begin{aligned} w_s^2 &= x_0 a\beta y + \left(1-x_0\right)b, \quad \text{(A.4)} \\ x_0 &= \frac{r+q+p_1+p_2}{p_2+2\left(r+q+p_1\right)}, \end{aligned}$$

$$\begin{aligned} w_n^2 &= xay + \left(1-x\right)b, \quad \text{(A.5)} \\ x &= \frac{r+q+p_2}{p_2+2\left(r+q\right)}, \end{aligned}$$

where we note that $x_0 < x$. Hence, we have:

$$w_s^2 - w_n^2 = \left(x_0 a\beta - x\right)y - \left(x_0 - x\right)b = \left(x_0 - x\right)\left(ay-b\right) + x_0\left(\beta-1\right)ay.$$

For $\beta \leq 1$ we have that $w_s^2 < w_n^2$. For $\beta > 1$ we have that $w_s^2 > w_n^2$ iff

$$\left(x_0 - x\right)\left(ay-b\right) + x_0\left(\beta-1\right)ay > 0,$$

which may be reduced to

$$\left(r+q+p_1+p_2\right)\left(p_2+2\left(r+q\right)\right)\left(\beta-1\right)ay > p_2 p_1\left(ay-b\right).$$

We have the sufficient condition:

$$\left(\beta-1\right)ay > ay - b,$$

Finally, we show that $w_s^1 > w_n^2$. Rewrite equation (A.1), using equation (A.4):

$$w_s^1 = x_1 y + \left(1-x_1\right)\left(b + \frac{p_2\left(a\beta y - b\right)}{p_2+2\left(r+q+p_1\right)}\right), \quad \text{(A.6)}$$

where $x_1 = \frac{\left(r+q+p_1\right)}{p_1+2\left(r+q\right)}$. As $x_1 > x$ for $p_1 > p_2$ we have from comparison of equation (A.5) and (A.6) that $w_s^1 > w_n^2$.

B. Appendix B

We want to show that the wage effects are:

$$\begin{aligned}
\frac{dw_n^2}{dp_2} &> 0, \frac{dw_n^2}{dp_1} = 0 \\
\frac{dw_s^2}{dp_2} &> 0, \frac{dw_s^2}{dp_1} < 0 \\
\frac{dw_s^1}{dp_2} &> 0, \frac{dw_s^1}{dp_1} > 0
\end{aligned}$$

Using equation (A.5) we have:

$$w_n^2 = xay + (1-x)\, b,\ \frac{dx}{dp_2} > 0,\ \frac{dx}{dp_1} = 0$$

Hence we have shown that:

$$\frac{dw_n^2}{dp_2} > 0 \text{ and } \frac{dw_n^2}{dp_1} = 0 \tag{B.1}$$

Use equation (A.4) we have:

$$w_s^2 = x_0 a\beta y + (1-x_0)\, b, \frac{dx_0}{dp_2} > 0, \frac{dx_0}{dp_1} < 0,$$

Hence we have shown that:

$$\frac{dw_s^2}{dp_2} > 0 \text{ and } \frac{dw_s^2}{dp_1} < 0. \tag{B.2}$$

Next, we know want to show that $\frac{dw_s^1}{dp_2} > 0$ and $\frac{dw_s^1}{dp_1} > 0$. Use equation (A.6) and we have:

$$\begin{aligned}
w_s^1 &= x_1 y + (1-x_1)\, b + x_3\, (a\beta y - b)\,, \\
x_3 &= (1-x_1)\, \frac{p_2}{p_2 + 2\,(r+q+p_1)}.
\end{aligned} \tag{B.3}$$

As $\frac{dx_3}{dp_2} > 0$ and $\frac{dx_1}{dp_2} = 0$, we have:

$$\frac{dw_s^1}{dp_2} > 0$$

As $\frac{dx_1}{dp_1} > 0$ and $\frac{dx_3}{dp_1} < 0$ we have two opposite effects on w_s^1 from p_1. However, the total impact on w_s^1 is:

$$\begin{aligned}
&\frac{\partial w_s^1}{\partial p_1} \\
= \;& \frac{\partial x_1}{\partial p_1}(y-b) + \frac{\partial x_3}{\partial p_1}(a\beta y - b) = \\
& (r+q)\frac{\left((1-a\beta)\,y\,(p_2+4\,(r+q+p_1))\,p_2 + (y-b)\,4\,(r+q+p_1)^2 - p_2 2\,(r+q)\,(a\beta y-b)\right)}{(p_1+2\,(r+q))^2\,(p_2+2\,(r+q+p_1))^2} \\
> \;& 0,
\end{aligned}$$

as $a\beta < 1$ and $p_1 > p_2$.

C. Appendix C

The equations giving p_1 as a function of p_2 and p_2 as a function of p_1 are:

$$\Phi = -k + \frac{f}{p_1}\left(\frac{y-w_s^1}{r+q}\right) = 0 \tag{C.1}$$

$$\begin{aligned}
\Psi &= -k + \frac{f}{p_2}\left(\frac{1}{1+\frac{\Lambda}{1-\Lambda}\frac{p_2+q}{p_1+p_2+q}}\left(\frac{ay-w_n^2}{r+q} - \frac{a\beta y - w_s^2}{r+q+p_1}\right) + \frac{a\beta y - w_s^2}{r+q+p_1}\right) \\
&= 0
\end{aligned} \tag{C.2}$$

where:

$$\lambda_N = \frac{1}{1+\frac{\Lambda}{1-\Lambda}\frac{p_2+q}{p_1+p_2+q}},\ \frac{d\lambda_N}{dp_1} > 0,\ \frac{d\lambda_N}{dp_2} < 0,\ \frac{d\lambda_N}{d\Lambda} < 0 \tag{C.3}$$

By implicit differentiation of equation (C.1) we get:

$$\frac{dp_1}{dp_2} \mid p_1(p_2) = -\frac{d\Phi/dp_2}{d\Phi/dp_1} = -\frac{-\frac{dw_s^1}{dp_2}}{-\frac{y-w_s^1}{p_1} - \frac{dw_s^1}{dp_1}} \tag{C.4}$$

As shown above the wage effects are positive, $\frac{dw_s^1}{dp_2},\ \frac{dw_s^1}{dp_1} > 0$, implying that $\frac{dp_1}{dp_2} \mid p_1(p_2) < 0$.

We want to go one step further and show that $0 > \frac{dp_1}{dp_2} \mid p_1(p_2) > -1$.

We established above that $\frac{dw_s^1}{dp_2} > 0$, hence rewriting equation (C.4) we get:

$$\frac{y-w_s^1}{p_1} + \frac{dw_s^1}{dp_1} - \frac{dw_s^1}{dp_2} > 0 \Rightarrow \frac{dp_1}{dp_2} \mid p_1(p_2) > -1 \tag{C.5}$$

Using equation (B.3) the reduced form of the inequality (C.5) is:

$$\frac{y-w_s^1}{p_1}+\frac{dw_s^1}{dp_1}-\frac{dw_s^1}{dp_2}>0\Leftrightarrow$$

$$\frac{(1-x_1)(y-b)-x_3(a\beta y-b)}{p_1}-(1-x_1)\frac{2(r+q+p_1)}{(p_2+2(r+q+p_1))^2}(a\beta y-b)$$
$$+\frac{dw_s^1}{dp_1}>0.$$

iff

$$\frac{(1-x_1)}{p_1}\frac{\left((y-b)(p_2+2(r+q+p_1))-\left(p_2+p_1\frac{2(r+q+p_1)}{p_2+2(r+q+p_1)}\right)(a\beta y-b)\right)}{p_2+2(r+q+p_1)}$$
$$+\frac{dw_s^1}{dp_1} > 0.$$

which is fulfilled as $a\beta \leq 1$. Hence we are able to conclude that:

$$0>\frac{dp_1}{dp_2}\mid p_1(p_2)>-1.$$

Turning to p_2 as a function of p_1 we get by implicit differentiation of equation (C.2):

$$\frac{dp_2}{d\Lambda} = -\frac{d\Psi/d\Lambda}{d\Psi/dp_2}=$$
$$\frac{-\frac{d\lambda_N}{d\Lambda}\left(\frac{ay-w_n^2}{r+q}-\frac{a\beta y-w_s^2}{r+q+p_1}\right)}{\frac{d\lambda_N}{dp_2}\left(\frac{ay-w_n^2}{r+q}-\frac{a\beta y-w_s^2}{r+q+p_1}\right)-\frac{\lambda_N\frac{ay-w_n^2}{r+q}+(1-\lambda_N)\frac{a\beta y-w_s^2}{r+q+p_1}}{p_2}-\frac{dw_n^2}{dp_2}\frac{\lambda_N}{r+q}-\frac{dw_s^2}{dp_2}\frac{1-\lambda_N}{r+q+p_1}}$$

As shown above the wage effects are positive and $\frac{d\lambda_N}{dp_2}<0$, $\frac{d\lambda_N}{d\Lambda}<0$. The denominator may be rewritten as

$$\left(\frac{d\lambda_N}{dp_2}-\lambda_N\right)\frac{ay-w_n^2}{r+q}+\left(-\frac{d\lambda_N}{dp_2}-\frac{(1-\lambda_N)}{p_2}\right)\frac{a\beta y-w_s^2}{r+q+p_1}-\frac{dw_n^2}{dp_2}\frac{\lambda_N}{r+q}-\frac{dw_s^2}{dp_2}\frac{1-\lambda_N}{r+q+p_1}.$$

A sufficient condition for a negative sign is that:

$$\left(-\frac{d\lambda_N}{dp_2}-\frac{(1-\lambda_N)}{p_2}\right)<0.$$

Rewrite the condition to obtain:

$$\left(\frac{\frac{\Lambda}{1-\Lambda}\frac{p_1}{(p_1+p_2+q)^2}}{\left(1+\frac{\Lambda}{1-\Lambda}\frac{p_2+q}{p_1+p_2+q}\right)^2} - \frac{\frac{\frac{\Lambda}{1-\Lambda}\frac{p_2+q}{p_1+p_2+q}}{1+\frac{\Lambda}{1-\Lambda}\frac{p_2+q}{p_1+p_2+q}}}{p_2} \right) = \frac{\frac{\Lambda}{1-\Lambda}\frac{1}{(p_1+p_2+q)}}{\left(1+\frac{\Lambda}{1-\Lambda}\frac{p_2+q}{p_1+p_2+q}\right)} \left(\frac{\frac{p_1}{p_1+p_2+q}}{1+\frac{\Lambda}{1-\Lambda}\frac{p_2+q}{p_1+p_2+q}} - \frac{p_2+q}{p_2} \right) < 0.$$

Hence the sign of $\frac{dp_2}{d\Lambda}$ depends on the sign of $\frac{ay-w_n^2}{r+q} - \frac{a\beta y-w_s^2}{r+q+p_1}$. We have that $\frac{dp_2}{d\Lambda} < 0$ if $\frac{ay-w_n^2}{r+q} - \frac{a\beta y-w_s^2}{r+q+p_1} > 0$. Rewrite the expression using the expression for w_s^2 equation (A.4), and the expression for w_n^2 equation (A.5):

$$\frac{ay-w_n^2}{r+q} - \frac{a\beta y-w_s^2}{r+q+p_1} =$$

$$\frac{ay-w_n^2}{r+q} - \frac{a\beta y-w_s^2}{r+q+p_1} = \frac{ay-b}{p_2+2(r+q)} - \frac{\beta ay-b}{p_2+2(r+q+p_1)} \tag{C.6}$$

A critical value of β is derived:

$$\begin{aligned} \frac{ay-b}{p_2+2(r+q)} - \frac{\beta ay-b}{p_2+2(r+q+p_1)} &> 0 \Rightarrow \\ \frac{a\beta y-b}{ay-b} &< \frac{p_2+2(r+q+p_1)}{p_2+2(r+q)} \Rightarrow \\ \beta &< 1+\frac{\frac{ay-b}{ay}2p_1}{p_2+2(r+q)} = \beta^* \end{aligned}$$

Hence, we have established that $\frac{dp_2}{d\Lambda} < 0$ for $\beta < \beta^*$. For $\beta > \beta^*$ the sign is positive: $\frac{dp_2}{d\Lambda} > 0$.

The slope of the function $p_2(p_1)$ is obtained by implicit differentiation of equation (C.2):

$$\frac{dp_2}{dp_1} = -\frac{d\Psi/dp_1}{d\Psi/dp_2} = \frac{-\left(\frac{d\lambda_N}{dp_1}\left(\frac{ay-w_n^2}{r+q}-\frac{a\beta y-w_s^2}{r+q+p_1}\right)-\frac{(1-\lambda_N)}{r+q+p_1}\left(\frac{dw_s^2}{dp_1}+\frac{a\beta y-w_s^2}{r+q+p_1}\right)\right)}{\frac{d\lambda_N}{dp_2}\left(\frac{ay-w_n^2}{r+q}-\frac{a\beta y-w_s^2}{r+q+p_1}\right)-\frac{\lambda_N\frac{ay-w_n^2}{r+q}+(1-\lambda_N)\frac{a\beta y-w_s^2}{r+q+p_1}}{p_2}-\frac{dw_n^2}{dp_2}\frac{\lambda_N}{r+q}-\frac{dw_s^2}{dp_2}\frac{1-\lambda_N}{r+q+p_1}}$$

We can show that this slope is greater than -1. As shown above the denominator is negative.

Hence, if the inequality:

$$\begin{aligned} & \frac{d\lambda_N}{dp_1}\left(\frac{ay-w_n^2}{r+q}-\frac{a\beta y-w_s^2}{r+q+p_1}\right)-\frac{(1-\lambda_N)}{r+q+p_1}\left(\frac{dw_s^2}{dp_1}+\frac{a\beta y-w_s^2}{r+q+p_1}\right) \quad \text{(C.7)} \\ - \quad & \frac{d\lambda_N}{dp_2}\left(\frac{ay-w_n^2}{r+q}-\frac{a\beta y-w_s^2}{r+q+p_1}\right)+\frac{\lambda_N\frac{ay-w_n^2}{r+q}+(1-\lambda_N)\frac{a\beta y-w_s^2}{r+q+p_1}}{p_2} \\ & +\frac{dw_n^2}{dp_2}\frac{\lambda_N}{r+q}+\frac{dw_s^2}{dp_2}\frac{1-\lambda_N}{r+q+p_1} \\ > \quad & 0, \end{aligned}$$

holds we have that:

$$\frac{dp_2}{dp_1} \mid p_2\left(p_1\right) > -1.$$

For $\beta < \beta^*$ we have:
By use of equation (C.3), (C.6) and (B.1) we know that:

$$\left(\frac{d\lambda_N}{dp_1}-\frac{d\lambda_N}{dp_2}\right)\left(\frac{ay-w_n^2}{r+q}-\frac{a\beta y-w_s^2}{r+q+p_1}\right)+\frac{\lambda_N\frac{ay-w_n^2}{r+q}}{p_2}+\frac{dw_n^2}{dp_2}\frac{\lambda_N}{r+q}>0$$

And as $p_1 > p_2$ we have that $\left(\frac{dw_s^2}{dp_2}-\frac{dw_s^2}{dp_1}-\frac{a\beta y-w_s^2}{r+q+p_1}+\frac{a\beta y-w_s^2}{p_2}\right)>0$. Hence, for $\beta < \beta^*$ we have established that:

$$\frac{dp_2}{dp_1} \mid p_2\left(p_1\right) > -1.$$

For $\beta > \beta^*$ we can show the following:

$$\frac{p_2+2\left(r+q+p_1\right)}{p_2+2\left(r+q\right)} < \frac{a\beta y-b}{ay-b} < \frac{T}{B}\frac{p_2+2\left(r+q+p_1\right)}{p_2+2\left(r+q\right)} \Rightarrow \frac{dp_2}{dp_1} \mid p_2\left(p_1\right) > -1. \quad \text{(C.8)}$$

where $\frac{T}{B}=\frac{\frac{\lambda_N(1-\lambda_N)}{p_2+q}+\frac{2\lambda_N(p_2+r+q)}{p_2(p_2+2(r+q))}}{\frac{\lambda_N(1-\lambda_N)}{p_2+q}-\frac{2(1-\lambda_N)(r+q+p_1)}{p_2(p_2+2(r+q+p_1))}}>1.$

The first inequality in equation (C.8) defines β^* and the second inequality defines $\hat{\beta}$ Hence if $\beta < \hat{\beta}$ where $\beta^* < \hat{\beta}$, we have established that: $\frac{dp_2}{dp_1} \mid p_2\left(p_1\right) > -1$.

D. Appendix D

The equations for the change in wage dispersions are:

$$\frac{\partial WD_2}{\partial \Lambda} = \frac{dw_n^2}{d\Lambda} - \frac{dw_s^2}{d\Lambda}, \tag{D.1}$$

$$\frac{\partial WD_{sn}}{\partial \Lambda} = \frac{dw_s^1}{d\Lambda} - \frac{dw_n^2}{d\Lambda}, \tag{D.2}$$

$$\frac{\partial WD_{ss}}{\partial \Lambda} = \frac{dw_s^1}{d\Lambda} - \frac{dw_s^2}{d\Lambda}. \tag{D.3}$$

Sign of $\frac{\partial WD_2}{\partial \Lambda}$
We have that

$$\frac{\partial WD_2}{\partial \Lambda} = \frac{dw_n^2}{d\Lambda} - \frac{dw_s^2}{d\Lambda} = \left(\frac{dx}{dp_2}(ay - b) - \frac{dw_s^2}{dp_2} - \frac{dw_s^2}{dp_1}\frac{\partial p_1}{\partial p_2} \right) \frac{\partial p_2}{\partial \Lambda},$$

which has the same sign as

$$-\frac{r+q}{\left(p_2 + 2(r+q)\right)^2}(ay - b) + \frac{(r+q+p_1)}{\left(p_2 + 2(r+q+p_1)\right)^2}(a\beta y - b) + \frac{dw_s^2}{dp_1}\frac{\partial p_1}{\partial p_2}.$$

which has an ambiguous sign for all β.
Sign of $\frac{\partial WD_{sn}}{\partial \Lambda}$
We have that equation (D.2) is:

$$\frac{\partial WD_{sn}}{\partial \Lambda} = \left(\frac{dw_s^1}{dp_1}\frac{\partial p_1}{\partial p_2} + (a\beta y - b)\frac{\partial x_3}{\partial p_2} - \frac{dx}{dp_2}(ay - b) \right) \frac{\partial p_2}{\partial \Lambda},$$

which for $\beta < \bar{\beta}$ has the same sign as $-\frac{\partial p_2}{\partial \Lambda}$.
$\bar{\beta}$ is determined by the equation:

$$\frac{a\beta y - b}{ay - b} = \frac{\left(p_1 + 2(r+q)\right)\left(p_2 + 2(r+q+p_1)\right)^2}{2(r+q+p_1)\left(p_2 + 2(r+q)\right)^2}.$$

It can be shown that $\beta^* < \bar{\beta}$.
Hence we the following:

$$\frac{\partial WD_{sn}}{\partial \Lambda} > 0 \text{ if } \beta < \beta^*,$$

$$\frac{\partial WD_{sn}}{\partial \Lambda} < 0 \text{ if } \beta^* < \beta < \bar{\beta}.$$

Otherwise the sign is indeterminate.
Sign of $\frac{\partial WD_{ss}}{\partial \Lambda}$
Equation (D.3) can be rewritten:

$$\frac{\partial WD_{ss}}{\partial \Lambda} = \left(\frac{dw_s^1}{dp_1}\frac{\partial p_1}{\partial p_2} + (a\beta y - b)\frac{\partial x_3}{\partial p_2} - \frac{dw_s^2}{dp_2} - \frac{dw_s^2}{dp_1}\frac{\partial p_1}{\partial p_2} \right) \frac{\partial p_2}{\partial \Lambda},$$

It can be shown that:

$$\text{sign}\left(\frac{\partial WD_{ss}}{\partial \Lambda}\right) = \text{sign}\left(-\frac{\partial p_2}{\partial \Lambda}\right).$$

Hence we have:

$$\begin{aligned} \frac{\partial WD_{ss}}{\partial \Lambda} &> 0 \text{ if } \beta < \beta^*, \\ \frac{\partial WD_{ss}}{\partial \Lambda} &< 0 \text{ if } \beta > \beta^*. \end{aligned}$$

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